Guidance to Support the Teaching of Written Calculations

We aim to ensure that by the end of year six the children will understand and successfully use compact written methods to carry out and record calculations that they cannot do in their heads.

To enable children to move towards compact written methods with full understanding, a step by step approach is taken. For each of the four operations children are first introduced to expanded methods that lead to a compact form of calculation. It is important that children feel secure and comfortable with each stage before they move on to the next.
Children will progress through the stages of expanded calculations at different rates. It is far better that they can operate efficiently at any stage and with understanding than to move them on too quickly.

Approach

The children need to approach any calculation by asking themselves the following questions:

- Can I do this in my head?
- Can I estimate the size of the answer?
- If I cannot do it wholly in my head, what do I need to write down in order to help me calculate the answer?
- Will the written method I know be helpful?
- Does my answer make sense?

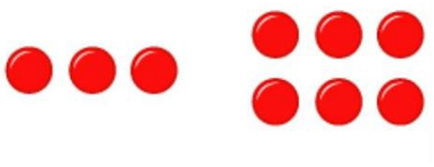
Whenever appropriate the children should do mental calculations. In order to support this approach calculations are always presented to children horizontally so they can make decisions as to the best way to solve the problem. When recording calculations vertically the operation sign should be placed on the left side of the calculation on the lower row.

Progress in Addition

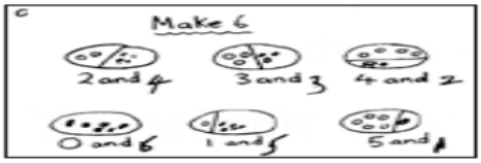
Step 1

Grouping objects

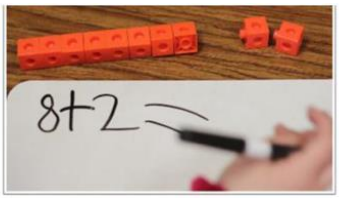
Children begin to add ones together by using physical objects, e.g. cups, counters, shapes, toys. They count each object to find out how many altogether whilst the teacher models the language.



Children begin to develop ways of recording their calculations using pictures.



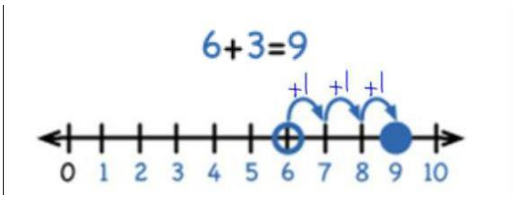
The teacher models what the adding of two groups looks like in a number sentence. The children begin to copy these number sentences onto whiteboards whilst still using objects to add.



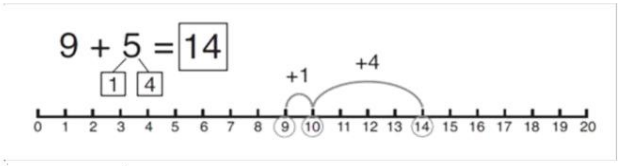
Step 2

Using number lines to add one digit numbers to one digit numbers.

Children are shown how to add using a pre-numbered number line starting with the biggest number. They record their findings orally to begin with before moving on to drawing the jumps themselves.
Note: Each jump is one.

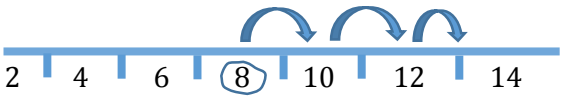


When children are ready, this will be extended to counting on in larger jumps. Children may want to jump to the nearest 10 and count on from there. Some children may not need to do that as they feel confident going through the tens boundary.

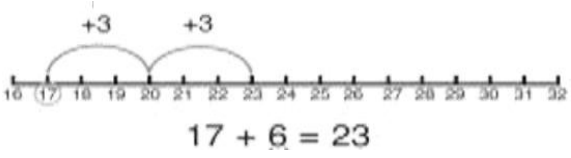


As children become more confident they move on to partially numbered number lines.

8 + 5 = 13



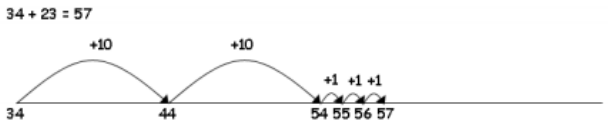
This can be extended to adding a one digit number to a two digit number.



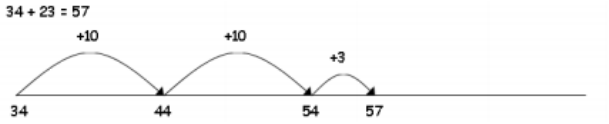
Step 3

Using number lines to add two digit numbers to two digit numbers.

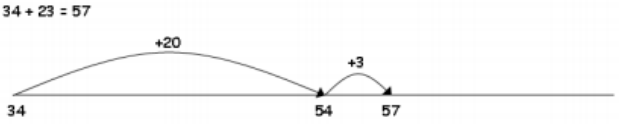
Children will begin to use empty number lines themselves starting with the largest number and counting on. They will use their knowledge of partitioning to support the jumps they make.



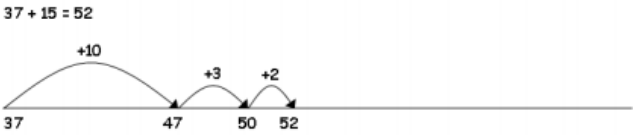
✓ Then helping children to become more efficient by adding the units in one jump (by using the known fact 4 + 3 = 7).



Children will then move onto moving in multiplies of 10 and then add on the ones.



✓ Bridging through ten can help children become more efficient.



Step 4

Using partitioning without a number line.

Children will build on their number knowledge by partitioning two digit numbers into tens and ones. They add the tens together and the ones together then recombine to find the total.
The children must be confident in their understanding of place value before moving on.
This step will allow the children to use partitioning to add the numbers mentally.

37 + 15

30 7 10 5

30 +10 = 40

7+ 5 = 12

40 + 12 = 52

Step 5

Children can use their partitioning knowledge to lay out their calculation in a vertical way. This prepares them for the next stage of column addition. The first example shows children using the method with no carrying.

Vertically 23 + 32 =

T	U
20	+ 3
30	+ 2

50 + 5 = 55

Children will then move on and begin to carry their units over.

Vertically 47 + 76 =

T	U
40	+ 7
70	+ 6

110 + 13 = 123

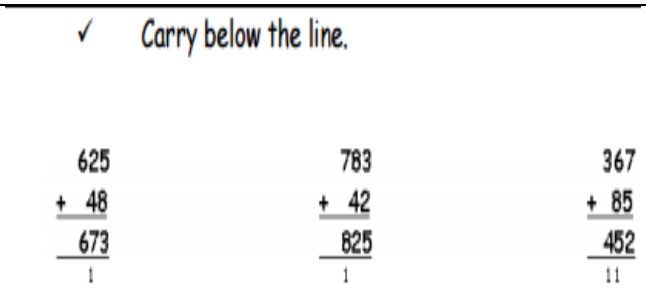
Once confident, children can start using the partitioning column method to solve problems that bridge the tens and hundreds boundaries.

337 + 188 = 525

H	**T**	**U**
300	+ 30 +	7
100	+ 80 +	8
400	+ 110 +	15 = 525

Children will be introduced to 'carrying.' which happens when bridging in the column method.
Please note:

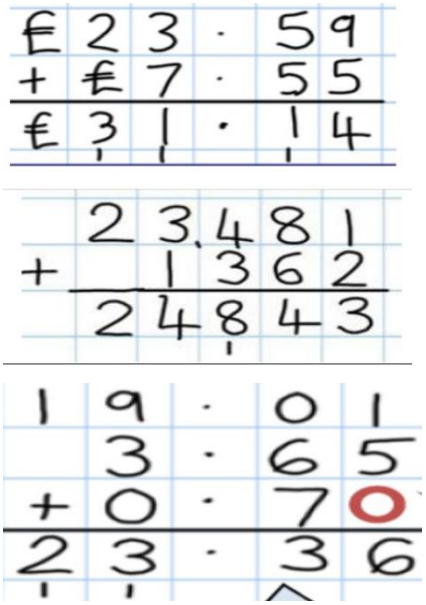
1) Ones must be added first.
2) 'Carry' numbers underneath the bottom line.
3) Reinforce the place value. It is not 1 it is 1 ten.



Step 7

Once children are confident with this method, children will:

- Begin to add two or more three-digit sums of money, with or without adjustment from the pence to the pounds;
- Know that the decimal points should line up under each other, particularly when adding or subtracting mixed amounts, e.g. £3.59 + 78p
- Add several numbers with different numbers of digits including tens of thousands
- Add decimals together using zeros as place holders to make it clearer. Always remembering to line up the decimal points



Key Vocabulary

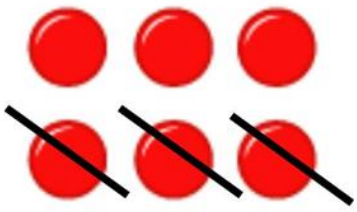
Add, more, plus, and, make, altogether, total, equal to, equals, the same as, double, most, count on, number line, sum, tens, units, partition, addition, column, tens boundary, hundreds boundary, increase, vertical, carry, expanded, compact

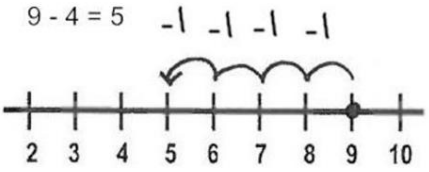
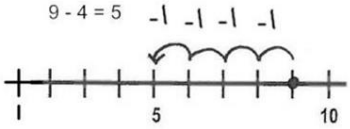
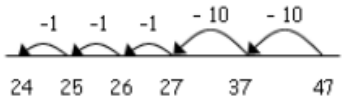
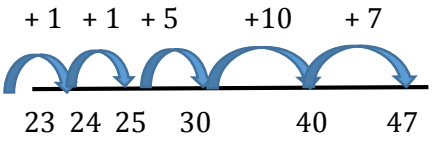
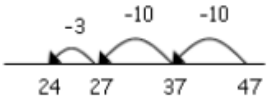
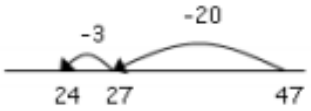
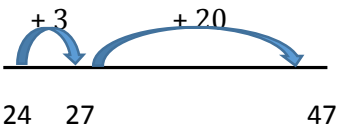
Progress in Subtraction

Step 1- Grouping and number sentences

Children begin to subtract units from a large group using physical objects, e.g counters and numicons.

They count each object to find out how many are left and begin to record by drawing pictures/marks.



The teacher models what the subtraction looks like in a number sentence.	
Step 2 **Using number lines to subtract one digit numbers** Children then begin to use numbered lines to support their own calculations - using a pre-numbered line to count back in ones from the biggest number The number line should also be used to show that 9 - 4 means the 'difference between 9 and 4' or 'the difference between 4 and 9' and how many jumps they are apart. Children will then move onto partially numbered lines.	 
Step 3 **Using number lines to subtract two digit numbers** Children will begin to use empty number lines to support calculations. First counting back in tens and ones. Please note that some children may find it easier to use a number line to add on from the smaller number to find the difference. Children can then, when feeling more confident, subtract the ones in a single jump. Followed by subtracting the tens in one jump and the ones in another. This can be extended to larger numbers. Please note: A number line is a good way to find the difference between numbers when they are closer together.	47 – 23 = 24  Or:  47 – 23 = 24   OR 

Step 4: **Expanded column subtraction** Children should have good partitioning and mental skills knowledge to approach the column method of subtraction. At first they should attempt it where no exchange is required.	T U 80 9 - 30 5 50 + 4
Step 5 **Column subtraction (no decomposition)** When children are confident with the expanded column and the understanding of why and how we subtract, they can compact the numbers with a vertical layout and no decomposition. Remember always put the larger number on top.	378 -126 252
Step 6 Children should be introduced to exchanging through practical tools. Base 10 or Dienes cubes are vital to show children that the value is 7 groups of ten and show the exchange of a ten to the ones so that the subtraction works. Children can then use this method to subtract larger amounts and it can be used to subtract money also.	 70+12 - 40+7 20+5 = 25 238 - 146 = 92 100 200 + 30 + 8 - 100 + 40 + 6 0 + 90 + 2
Step 7 **Column subtraction with decomposition.** Children then exchange using the compact method of subtraction. Using their knowledge from prior stages, they will be able to see how to exchange between columns. Children will use the compact method to solve problems involving numbers up to six digits and beyond and solve problems where they will need to 'exchange several times.' They will also solve problems involving decimals and continue to apply their knowledge of decimal place value to line up the decimal points. They will use zeros as place holders.	2 11 1 3 2 6 1 7 8 1 4 8 6 8 10, 6 9 9 - 8 9, 9 4 9 6 0, 7 5 0 5 10 15 . 3 11 9 - 3 6 . 0 8 0 6 9 . 3 3 9

Key Vocabulary

Equal to, take, take away, less, minus, subtract, leaves, distance between, difference between, how many more, how many fewer/less than, most, least, count back, how many left, how much less is_?, count on, strategy, partition, tens, units, exchange, decrease, hundreds, value, digit, inverse, tenths, hundredths, decimal place, decimal.

Progress in Multiplication

Step one

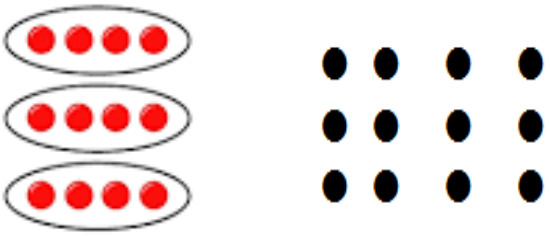
The teacher gives verbal instructions showing children how to 'multiply' the same amount of objects. E.g. 'I give out 3 sweets and I do the same 4 times'. The children record pictorially.

The written multiplication sentence will be modelled by the children.

Children will use pictorial representation or written description.

This could be shown as three groups of four. Children could draw the required amount of groups and fill them in, counting the total. Some children may draw four groups of three.

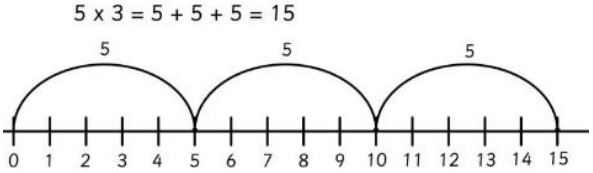
3 x 4 = or 4 x 3 =

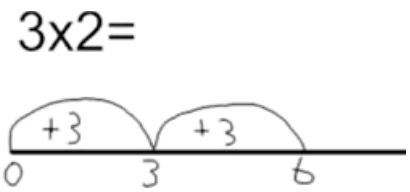


Step 2

Repeated addition moves the learning on and children can use a pre-numbered number line to multiply groups of numbers then move onto a partially numbered or blank number line once they are more confident.

Children will continue to use this method to multiply one digit by one digit numbers.
It is important to note that children, by the end of year two, should be able to recall multiplication facts for the twos, fives and tens and be able to count up in steps of three.





Step 3

Partitioning to multiply

Using mental methods to support multiplication for two digits by one digit.
Using partitioning, children begin to confidently mentally multiply ten by a one (unit) and then a one by a one. They then add and recombine the total together.

Children will know that 3 x 6 = 18 so 30 x 6 = 180. This allows for children to have good place value knowledge which supports them in their later stages of multiplication.

36 x 6 =

30 6

30 x 6 = 180
6 x 6 = 36
360 + 24 =216

Step 4

The grid method.

The grid method builds on the process of using partitioning and place value. The largest number is multiplied first in order to get a sense of the size of the number. Then you continue to multiply and add the multiplied totals together.
This is why it is important to have such good place value knowledge.

×	30	5
7	210	35

210 + 35 = 245

The grid method is extended so that children will now multiply 3 digit numbers by 1 digit numbers. When adding the 3 answers up to create a total, column addition could be used to ensure accuracy, especially where bridging will be needed

x	600	10	3
5	3000	50	15

Add up 3000, 50 and 15 to make 3065.

613 x 5 = 3065

This could be applied to two digit by two digit numbers.
You make the grid longer to accommodate the two numbers. The same method still applies and you still partition the digits.

The total would need to be added.

57 x 26 =

X	50	7
20	1000	140
6	?	

Step 5

Multiplication by a single digit by two digits.

The compact short multiplication can be tricky and needs to be modelled carefully by teachers.
It is important that children understand that they are multiplying 7 lots of 8 and then 7 lots of 30 and not 3.
It can be modelled like the picture across the page by the teacher to ensure children know why they are doing the process of the multiplication.

If necessary, the teacher may feel it would be useful for children to practice this written method (including putting the brackets at the side) before moving on.

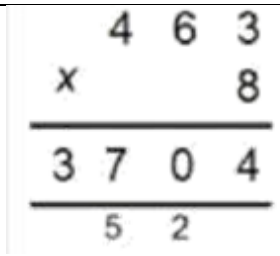
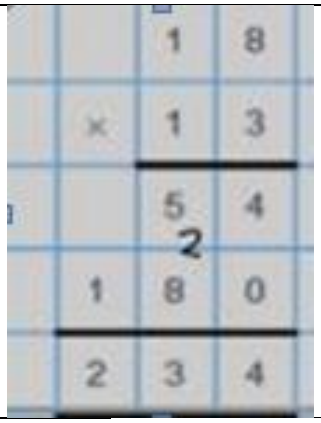
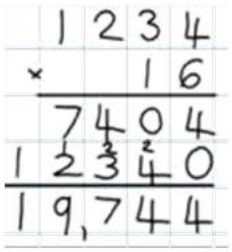
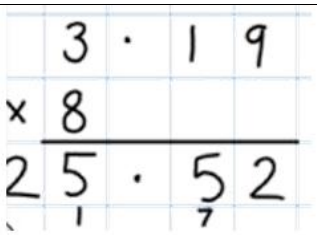
38
x 7
56 (7 x 8)
210 (7 x 30)
266

Depending on the child, they may be able to move straight onto the compact short multiplication method.

In this method children will carry their tens onto the column and add them on to the next calculation.

It is at this stage that approximation and estimation should become a regular part of classroom practice. Children should

34 x 6
H T U
34
x 6
204
2

approximate an answer before using a method so they know if their answer is accurate or not. 253 x 9 is approximately 250 x 10 = 2500 When children are confident with two digits by one digit, they can move onto three digits.	 463 x 8 = 3704 5 2
<u>Multiplying two digits by two digits.</u> Teacher model: It becomes more tricky when children are required to multiply two digits by two digits. Similarly to before, the teacher needs to ensure the children understand the process of why each part of the multiplication process is taking place. The teacher may model this example to ensure the children understand this before moving them on to the compact method of long multiplication.	56 x 27 42 (7 x 6) 350 (7 x 50) 120 (20 x 6) 1000 (20 x 50) 1512
<u>Step 6</u> **<u>Compact method of multiplication.</u>** In this stage children complete the first stage, 8 x 3 and 10 x 3 on the first row (carrying any tens digits over if necessary). Then they move the second row, placing a 0 at the end to show we are multiplying by ten and complete 8 x 1, 10 x 1. Then they add the two together to get the combined answer.	 18 × 13 = 54 + 180 = 234
Once long multiplication methods are secure, children are ready to move on to more challenging problems which require greater levels of mental calculation. The problem to the right shows 1234 x 6 on the top line and 1234 10 on the bottom line and the total of both calculations on the final row.	 1234 × 16 = 7404 + 12340 = 19,744
Consolidating short and long multiplication by multiplying decimals by one digit. When multiplying decimals it is important to remember that the digit you are multiplying by needs to be lined up with the ones digits. As with all decimal work, the decimal points must be lined up and the children need to have a clear understanding why that is.	 3.19 × 8 = 25.52
Key Vocabulary Groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, commutative, sets of, equal groups, times as big as, once, twice, three times, partition, grid method, multiple, product, tens, units, value, inverse, *square, factor, integer, decimal, short/long multiplication, carry*	

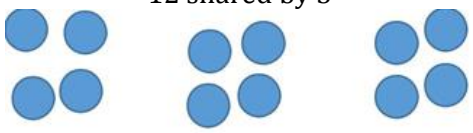
Progress in Division

Step 1

The children will start by physically sharing objects between set groups. E.g 12 sweets shared between three children.
They will discuss how to share equally so no group has more or less.

The division sentence will be modelled by the teacher and the children will start to copy onto whiteboards/ into their books.

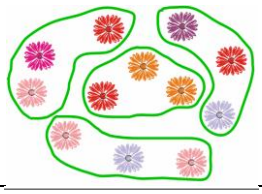
12 shared by 3



$12 \div 3 = 4$

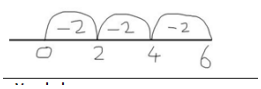
Children will begin to move away from physical objects and start to draw groups instead.
For the calculation, 12 ÷ 4 = 3, children draw how many groups they are sharing between (4) and fill in equally the groups until they have reached 12. They can then count how many are in each group.

12 ÷ 4 = 3

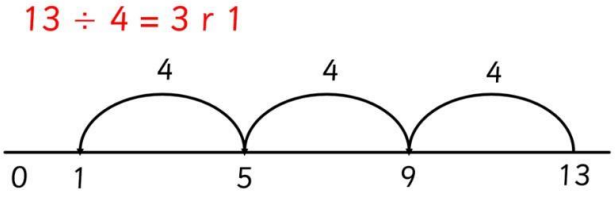


Step 2

When the children are confident at using arrays to group for division, they will move onto number lines to do repeated subtraction.



This can be used to show divisions where there may also be remainder. These calculations can be checked and compared to grouping or array methods.



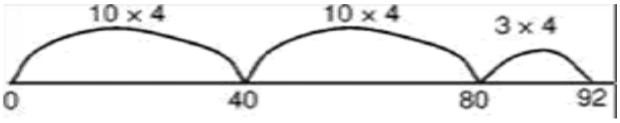
Whilst it is important for children to learn written methods, children will also be encouraged to use their times table knowledge to work out division calculations. This will help them with future division written calculations.

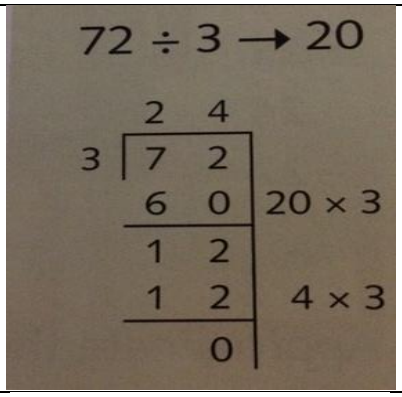
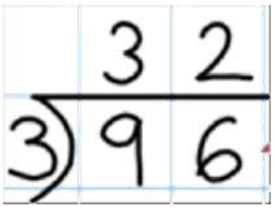
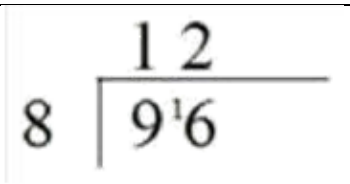
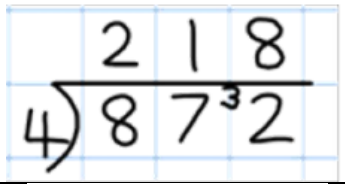
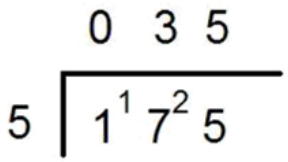
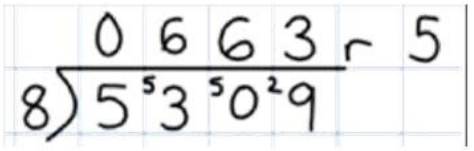
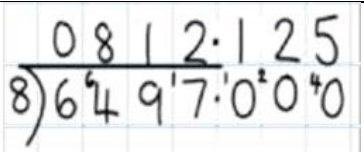
97 ÷ 9

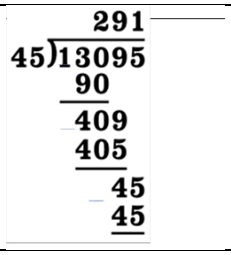
We know 10 x 9 = 90 with a remainder of 7

97 ÷ 9 = 10 r 7

Once confident, children will begin to solve problems on a grouping number line involving bigger numbers. To solve this effectively they will need to subtract chunks of the divisor. As you can see in the image for 92 ÷ 4, a step of 10 groups of 4 has been jumped, followed by another step of 10 jumps, and finally followed by a step of 3 jumps of 4. This means that in total 4 was jumped 23 times making 23 the answer.



Step 3 Expanded written method. This method of subtracting chunks can be shown in a different layout to begin to transition children into a more formal layout for division. The same process applies as above; children subtract known chunks away until they can't get anymore and count the groups of chunks they took away. This is why it is so important for children to have a secure knowledge of times tables.	
Step 4 Once children are confident with number line methods then they should start work on short division. Place value should be regularly discussed so children realise that they are partitioning and dividing the units then the tens by the divisor. Please Note: Initially children will start with simple problems where each digit is a multiple of the divisor.	
Once confident with the method of short division, they will move onto problems where the first digit of the dividend is not a multiple of the divisor and therefore a remainder will need to be carried. Children may need to use other equipment to calculate the division and multiplication facts required. Once children are secure, they can move onto three digit numbers.	 
Once children are confident at dividing with 3 digits, they need to attempt problems where the answer in the first column (hundreds column) is a zero. They may wish to record the hundred initially as this will help them remember its place and the numbers value	
Step 5 In year 5 children will begin to solve division problems where a number up to 4 digits is divided by a single digit number including answers with remainders. These division problems need to be contextual so the children learn how to express the remainder- as a number, a fraction, a decimals, rounded up or rounded down.	 or $\frac{5}{8}$
Step 6 As children get more secure the focus is on how, in short division, the remainders are expressed. Children need to express remainders as fractions and decimals, depending on the context.	

To divide by 2 digit numbers, the children will use the method of long division. This is when children subtract chunks of the calculation away in order to gain the answer.	

Key Vocabulary

Share, share equally, one each, two each…, group, groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, carry, remainder, multiple, divisible by, factor, quotient, prime number, prime factors, composite number (non-prime), *common factor*