

Keston Primary School



Guidance to Support the Teaching of Written Calculations

We aim to ensure that by the end of year 6 the children will understand and use successfully compact written methods to carry out and record calculations that they cannot do in their heads.

To enable children to move towards compact written methods with full understanding, a step by step approach is taken. For each of the four operations children are first introduced to expanded methods that lead to a compact form of calculation. It is important that children feel secure and comfortable with each stage towards compact methods before they move on to the next. Children will progress through the stages of expanded calculations at different rates. It is far better that they can operate efficiently at any stage and with understanding than to move them on too quickly.

Approach

The children need to approach any calculation by asking themselves the following questions:

- Can I do this in my head?
- Can I estimate the size of the answer?
- If I cannot do it wholly in my head what do I need to write down in order to help me calculate the answer?
- Will the written method I know be helpful?
- Does my answer make sense?

Whenever appropriate the children should do mental calculations. In order to support this approach calculations are always presented to children horizontally so they can make decisions as to the best way to solve the problem.

When recording calculations vertically the operation sign should be placed on the left side of the calculation on the lower row.

Addition and Subtraction

Criteria, which would indicate a child's readiness for formal written methods of addition and subtraction, would include:

- Knowledge of addition and subtraction facts to 20.
- Understanding of place value and ability to partition numbers into hundreds, tens and units.
- Understanding of commutative and associative laws of addition (though not of these terms).
- Ability to add at least three single digit numbers mentally.
- Ability to add and subtract any pair of two digit numbers mentally.
- Ability to explain mental strategies, orally and in writing.

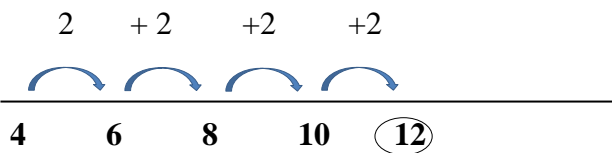
Progression in Addition

The following stages show a progression from informal methods for addition leading to a compact method.

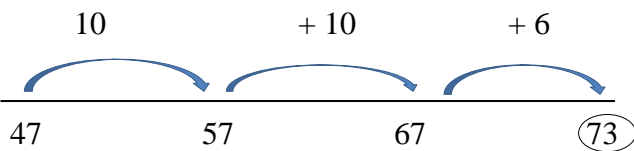
Stage 1

Numbers of varying size using a number line increasing the size of the steps

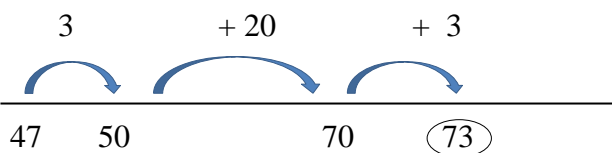
$$4 + 8 = 12$$



$$47 + 26 = 73$$



$$47 + 26 = 73$$



Stage 2

Using Partitioning without a number line

Horizontally $47 + 76 =$

$$40 + 70 = 110$$

$$7 + 6 = 13$$

$$110 + 13 = 123$$

Vertically $47 + 76 =$

$$\begin{array}{r} 40 + 7 \\ 70 + 6 \\ \hline \end{array}$$

$$110 + 13 = 123$$

Stage 3

Expanded vertical layout of a calculation showing the addition of the units and tens separately. Change to recording the smallest number first.

$$\begin{array}{r} 47 \\ + 76 \\ \hline 13 \\ 110 \\ \hline 123 \end{array}$$

Stage 4

Introduce calculations that involve 'carrying'.

$$\begin{array}{r} 47 \\ + 76 \\ \hline 123 \\ \hline 11 \end{array}$$

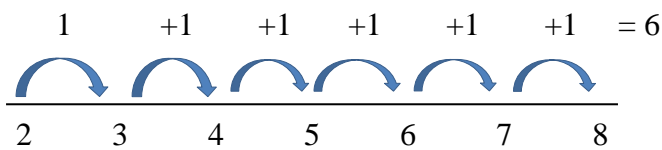
Use Stage 5 with larger numbers and decimals. Children may need to revert to more expanded layouts initially.

Progression in Subtraction

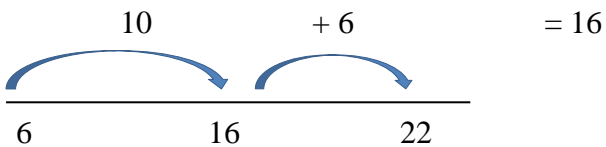
Stage 1

Using a number line – counting on to find the difference

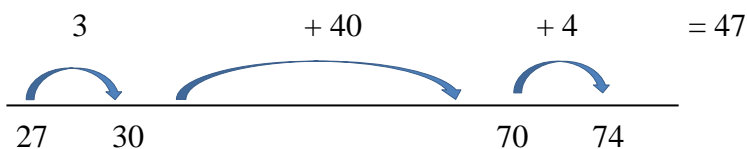
$$8 - 2 = 6$$



$$22 - 6 =$$



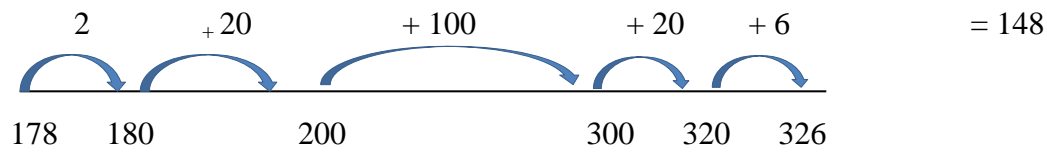
$$74 - 27$$



Stage 2

Using a number line with larger numbers and estimation – counting on to find the difference

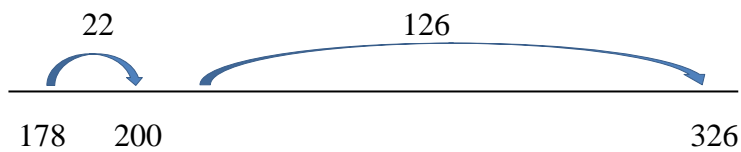
$$326 - 178$$



Stage 3

Using a number line with larger steps. Estimate first.

$$326 - 178$$



Stage 4

Introduce expanded layout with partitioning, lining up vertically, and no decomposition.

$$378 - 126$$

$$\begin{array}{r} 300 & 70 & 8 \\ -100 & 20 & 6 \\ \hline 200 & + & 50 & + & 2 & = & 252 \end{array}$$

Stage 5

Compact vertical layout with no decomposition, working from the least significant digit first in readiness for decomposition.

$$\begin{array}{r} 378 \\ -126 \\ \hline 252 \end{array}$$

Stage 6

Introduce decomposition modelling exchanging.

$$36 - 17 =$$

$$\begin{array}{r} 20 & 16 \\ \cancel{30} & \cancel{6} \\ -10 & 7 \\ \hline 10 & + & 9 & = & 19 \end{array}$$

Stage 7

Compact method with decomposition

$$\begin{array}{r} 211 \\ \cancel{3}26 \\ - 178 \\ \hline 148 \end{array}$$

Use the same methods for larger numbers and decimals

Multiplication and Division

Criteria, which would indicate a child's readiness for formal written methods of multiplication and division, include:

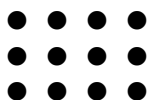
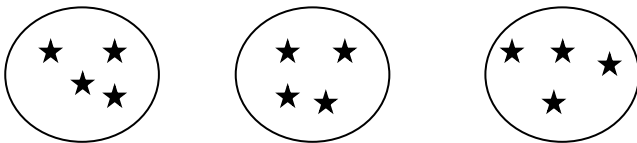
- Recall of multiplication and corresponding division facts for 2, 3, 4, 5 and 10 times tables.
- Understanding of what happens when a number is multiplied by 0 or 1.
- Understanding of place value.
- Understanding of 0 as a place holder.
- Ability to multiply two and three digit numbers mentally by 10 and 100.
- Ability to approximate products and quotients using powers of 10.
- Ability to double and halve two digit numbers mentally.
- Ability to use multiplication facts to derive mentally new multiplication facts.
- Ability to explain mental strategies, orally and in writing.

Progression in Multiplication

Stage 1

Using pictorial representation or written description

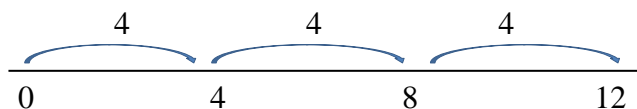
$$3 \times 4 =$$



Stage 2

Using a number line, showing repeated addition.

$$3 \times 4 =$$



Stage 3

Using partitioning, a mental method, which forms a basis for future more compact methods.

$$\begin{aligned} 38 \times 7 &= (30 \times 7) + (8 \times 7) \\ &= 210 + 56 \\ &= 266 \end{aligned}$$

Stage 4

The grid method which introduces partitioning using place value. The largest number is multiplied first in order to get a sense of the size of the answer.

$$38 \times 7 =$$

x	30	8	
7	210	56	266

Extend grid method to larger numbers and multiplication by two digit numbers.

$$56 \times 27 =$$

x	50	6	
20	1000	120	1120
7	350	42	392
			1512

Stage 5

Multiplication by a single digit, using expanded working in a vertical format. The least significant digit is recorded first. The children should be able to link this format with the grid method.

$$\begin{array}{r} 38 \\ \times 7 \\ \hline 56 \text{ (7 x 8)} \\ \underline{210} \text{ (7 x 30)} \\ \hline 266 \end{array}$$

Stage 6

Multiplication by a single digit, using vertical format and compact working.

$$\begin{array}{r} 38 \\ \times 7 \\ \hline 266 \\ 5 \end{array}$$

Stage 7

Multiplication by a two digit number using an expanded vertical format. Recording the unit digit first.

$$\begin{array}{r} 56 \\ \times 27 \\ \hline 42 \quad (7 \times 6) \\ 350 \quad (7 \times 50) \\ 120 \quad (20 \times 6) \\ \underline{1000} \quad (20 \times 50) \\ 1512 \end{array}$$

Stage 8

Compact method recording the smallest digit first. The children may need to use jottings to support multiplication.

$$\begin{array}{r} 56 \\ \times 27 \\ \hline 392 \quad (56 \times 7) \\ \underline{1120} \quad (56 \times 20) \\ 1512 \\ 1 \end{array}$$

Use the same methods for larger numbers and decimals.

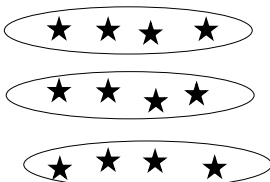
Progression in Division

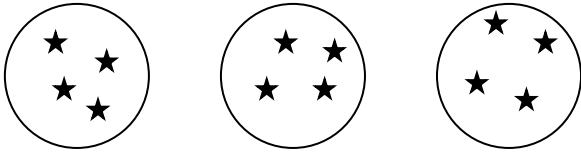
Stage 1

Before introducing expanded methods division may be recorded in a number of ways to facilitate understanding.

Sharing and grouping

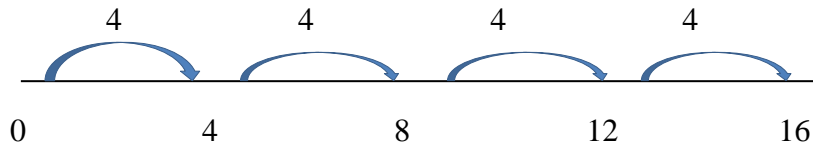
$$12 \div 3$$





Recording on a number line

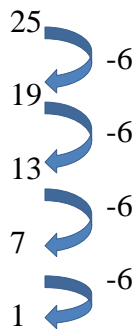
$$16 \div 4$$



Stage 2

Children are shown that division can be completed by repeated subtraction.

$$25 \div 6$$



I took away 4 lots of 6 and had one left over.

Stage 3

Using known facts to arrive at an answer.

$$97 \div 9$$

We know $10 \times 9 = 90$ with a remainder of 7

$$97 \div 9 = 10 \text{ r } 7$$

Stage 4

Introduce vertical layout and the symbol for division. $\overline{\hspace{1cm}}$

$$97 \div 9$$

$$\begin{array}{r} 9 \overline{)97} \\ \underline{90} \quad (9 \times 10) \\ 7 \end{array}$$

Answer 10 r 7

Stage 5

Using vertical layout, introduce larger numbers with the children subtracting in larger chunks provided they are comfortable with doing so. Answers should be estimated before the calculation.

$$\begin{array}{r} 6 \overline{)196} \\ \underline{-120} \quad (6 \times \underline{20}) \\ 76 \\ \underline{-60} \quad (6 \times \underline{10}) \\ 16 \\ \underline{-12} \quad (6 \times \underline{2}) \\ 4 \end{array}$$

The number which is used to multiply (6) is also the divisor

$$32 \text{ r } 4$$

$$\begin{array}{r} 6 \overline{)196} \\ \underline{-180} \quad (6 \times \underline{30}) \\ 16 \\ \underline{-12} \quad (6 \times \underline{2}) \\ 4 \end{array}$$

$$32 \text{ r } 4$$

Stage 6

Introduce the compact method, using zero as a place holder.

$$\begin{array}{r} 32 \text{ r } 4 \\ 6 \overline{)196} \\ \underline{-180} \\ 16 \\ \underline{-12} \\ 4 \end{array}$$

$$6 \overline{)196} \quad 32 \text{ r } 4$$

Use the same methods for larger numbers and decimals.